

OKLAHOMA STATE UNIVERSITY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING



ECEN 4503
Random Signals and Noise
Spring 2004
Final Exam



Choose any four out of five problems,
Please specify
1) _____; 2) _____; 3) _____; 4) _____;

Name : _____

Student ID: _____

E-Mail Address: _____

Problem 1:

Suppose the depth of water, measured in meters, behind a dam is described by an exponential random variable having a density

$$f_X(x) = \frac{1}{13.5} u(x) \exp\left(-\frac{x}{13.5}\right).$$

There is an emergency overflow at the top of the dam that prevents the depth from exceeding 40.6 m. There is a pipe placed 32.0 m below the overflow (ignore the pipe's finite diameter) that feeds water to a hydroelectric generator.

- a) What is the probability that water is wasted through emergency overflow?
- b) Given that water is not wasted in overflow, what is the probability the generator will have water to drive it?

Problem 2:

A random variable X is uniformly distributed on the interval $(-\pi/2, \pi/2)$. X is transformed to the new random variable $Y = T(X) = a \sin(X)$ with $a > 0$. Find the probability density function of Y .

Problem 3:

Given two random variables X and Y , find the probability density function of the random variable $Z = X/Y$ in terms of $f_X(x)$ and $f_Y(y)$.

Problem 4:

Given the function

$$f_{X,Y}(x,y) = \begin{cases} b(x+y)^2, & -2 < x < 2 \text{ and } -3 < y < 3 \\ 0, & \text{elsewhere} \end{cases}$$

- a) Find the constant b such that this is a valid joint density function.
- b) Determine the marginal density function $f_X(x)$.

Problem 5:

Let X and Y be statistically independent random variables with $E[X] = \frac{3}{4}$, $E[X^2] = 4$, $E[Y] = 1$, and $E[Y^2] = 5$. For a random variable $W = X - 2Y + 1$ find

a) R_{XY} , b) R_{YW} , and c) C_{XY} . In addition, d) are X and Y uncorrelated?